

High energy resummation for rapidity distributions^{*}

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Abstract

We discuss the generalisation of high-energy resummation to rapidity distributions to leading logarithmic accuracy. We test our procedure applying it to Higgs production in gluon-gluon fusion both with finite top mass and in the infinite mass limit. We check that they reproduce the known results at fixed order and we estimate finite top mass corrections to the NLO distribution.

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1 Introduction

The resummation of leading high energy (or small- x) contributions to hard QCD processes has been known for a long time in the case of inclusive cross-sections: heavy quark photo- and lepto-production [1], Deep-Inelastic Scattering [2], and more recently hadroproduction processes, including heavy quarks [3], Standard Model [4–6] and pseudo-scalar [7] Higgs production in gluon-gluon fusion, Drell-Yan [8] and prompt-photon [9]. In Ref. [10] we generalised the resummation formalism to rapidity distributions, opening the possibility to studies which are more relevant from a phenomenological viewpoint.

Such a result was made possible by a different, but equivalent, approach to small- x resummation. In the standard approach [1] one writes the cross-section in the high-energy limit as a convolution of a two-gluon irreducible hard part and a reducible ladder part, as part shown in Fig 1, on the left. This is the so called k_T -factorisation theorem:

$$\sigma(x, Q^2) = \int \frac{dz}{z} \frac{d\mathbf{k}_T^2}{\mathbf{k}_T^2} C\left(\frac{x}{z}, \frac{Q^2}{\mathbf{k}_T^2}\right) G(z, \mathbf{k}_T^2), \quad (1)$$

where C is interpreted as an off-shell coefficient function, while G is the k_T -dependent gluon Green's function. The resummation of small- x logarithms is usually obtained by taking G as the solution of the BFKL equation. More specifically, one diagonalises the convolution in Eq. (1) by computing Mellin moments with respect to x and Q^2 , obtaining

$$\Sigma(N, M) = h(N, M) F(N, M), \quad (2)$$

where we have defined

$$h(N, M) = M \int_0^1 dx x^{N-1} \int_0^\infty d\mathbf{k}_T^2 (\mathbf{k}_T^2)^{M-1} C(x, \mathbf{k}_T^2) \quad (3)$$

and F is the Mellin transformed of G (divided by M). The evolution of F then gives the pole condition $M = \gamma_s\left(\frac{\alpha_s}{N}\right)$, where $\gamma_s = \sum_k c_k \left(\frac{\alpha_s}{N}\right)^k$ is the BFKL anomalous dimension, which resums poles in N , i.e. logarithms of x . The resummed result in N space is then given by:

$$\Sigma(N) = h\left(N, \gamma_s\left(\frac{\alpha_s}{N}\right)\right). \quad (4)$$

In our approach we still start from Eq. (1) but rather than solving the BFKL equation for G , guided by the generalised ladder expansion for collinear factorisation [12], we look at G as the iteration of collinear safe kernels γ , as depicted in Fig. 1, on the right. We compute the cross-section for n insertions of the kernel γ and we subtract the first $n-1$ poles according to the $\overline{\text{MS}}$ prescription. We find:

$$\begin{aligned} \sigma_n\left(N, Q^2, \alpha_s \left(\frac{\mu^2}{Q^2}\right)^\epsilon, \epsilon\right) &= \gamma\left(N, \alpha_s \left(\frac{\mu^2}{Q^2}\right)^\epsilon, \epsilon\right) \int_0^\infty \frac{d\xi_n}{\xi_n^{1+\epsilon}} C\left(N, \xi_n, \alpha_s \left(\frac{\mu^2}{Q^2}\right)^\epsilon, \epsilon\right) \times \\ &\quad \frac{1}{(n-1)!} \frac{1}{\epsilon^{n-1}} \left[\sum_i \frac{\gamma_i(N, \alpha_s, 0)}{i} \left(1 - \left(\frac{\mu^2}{Q^2 \xi_n}\right)^{i\epsilon} \frac{\gamma_i(N, \alpha_s, \epsilon)}{\gamma_i(N, \alpha_s, 0)}\right) \right]^{n-1}, \end{aligned} \quad (5)$$

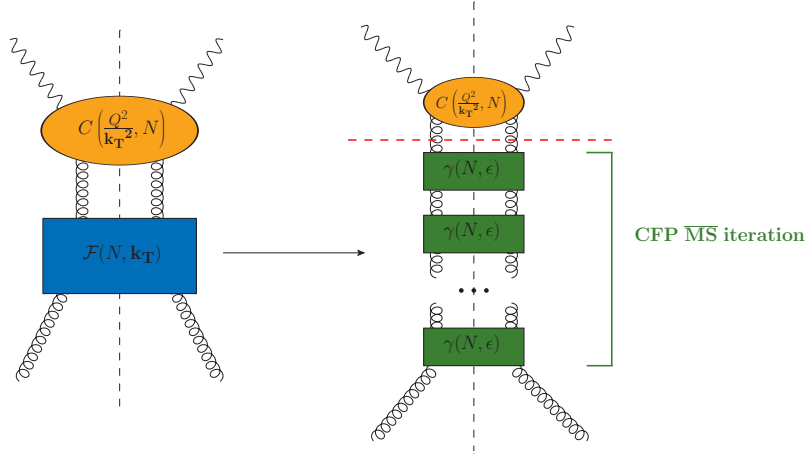


Figure 1: Left: decomposition of the partonic cross-section in terms of two-gluon irreducible hard part and a reducible ladder part. Right: generalized ladder expansion of the ladder part.

where $\xi = \frac{k_T^2}{Q^2}$. The full result is the obtained summing over n

$$\sigma(N, Q^2, \alpha_s) = \sum_n \sigma_n = \gamma(N, \alpha_s) \int_0^\infty d\xi \xi^{\gamma(N, \alpha_s)-1} C(N, \xi, Q^2, \alpha_s) R(N, \alpha_s). \quad (6)$$

This is the same as the one in Eq. (4), the only difference being the scheme dependent factor R , which reflects the fact that the calculation has been performed in $\overline{\text{MS}}$. The non-trivial information is encoded in the kernel γ . This kernel is an anomalous dimension, in the sense that it is the residue of a collinear pole. High-energy resummation is then achieved by choosing γ to be the dual [11] of the BFKL kernel. At the first non trivial order then $\gamma = \gamma_s$.

The main ingredient for obtaining Eq. (5) is the use of the same kinematics as in the proof of collinear factorisation [12]. Within this kinematics, the dependence on transverse and longitudinal momentum components are kept separate from each other, and this rendered the generalisation to rapidity distributions possible.

A simple expression for the all-order rapidity distribution in the high energy limit can be obtained for the Fourier-Mellin transformed of the partonic rapidity distribution:

$$\frac{d\sigma}{dy}(N, b) \equiv \int dx x^{N-1} \int dy e^{iby} \frac{d\sigma}{dy}(x, y). \quad (7)$$

The resummed result is then a simple generalisation of the inclusive case

$$\begin{aligned} \frac{d\sigma}{dy}(N, b) &= \int_0^\infty d\xi_1 \gamma_s \left(N + \frac{ib}{2} \right) \xi_1^{\gamma_s(N + \frac{ib}{2})-1} \times \\ &\times \int_0^\infty d\xi_2 \gamma_s \left(N - \frac{ib}{2} \right) \xi_2^{\gamma_s(N - \frac{ib}{2})-1} C(N, \xi_1, \xi_2, b). \end{aligned} \quad (8)$$

We note that the argument of the anomalous dimension γ_s is shifted by $\pm ib/2$. The off-shell coefficient function C is now differential in the partonic rapidity.

2 Higgs rapidity distribution

We have applied the formalism described in the previous section to Higgs production in gluon-gluon fusion. High-energy factorisation can be used to compute the small- x behaviour of the coefficient function keeping the full top mass dependence. By matching this result to an asymptotic expansion at large x , finite top mass effects have been evaluated to NNLO for the inclusive cross-section [13–15]. In the same spirit we construct an approximate NLO rapidity distribution by matching the result determined from high-energy factorisation with full top mass dependence at small- x to the one obtained in the infinite top mass limit at large x [16]. Before doing that we can use the analytic results for the NLO rapidity distribution in the heavy top limit to check our method. We apply our main result Eq. (8) to the case of Higgs production; expanding it to NLO and inverting the Fourier-Mellin transform, we find

$$\frac{d\sigma}{du}(x, u) = 3\sigma_0 \frac{\alpha_s}{\pi} \left[\frac{1}{(u-x)_+} - \delta(u-x) \ln x + \left(u \leftrightarrow \frac{1}{u} \right) \right] \quad \text{with} \quad u = e^{-2y}, \quad (9)$$

which is in full agreement with Ref. [16]. Similarly we can compute the small- x behaviour of the rapidity distribution keeping the top mass finite:

$$\frac{d\sigma}{du} = \sigma_0(\tau) c_1(\tau) \delta(u-x) + \left(u \leftrightarrow \frac{1}{u} \right) \quad \text{with} \quad \tau \equiv \frac{4m_t^2}{m_h^2}, \quad (10)$$

where c_1 has been determined numerically [10].

We study finite top mass effects by computing

$$R = \left(\frac{1}{\sigma_{NLO}} \frac{d\sigma_{NLO}}{dY} \right)_{\text{matched}} \bigg/ \left(\frac{1}{\sigma_{NLO}} \frac{d\sigma_{NLO}}{dY} \right)_{m_t \rightarrow \infty}. \quad (11)$$

Our findings are plotted in Fig. 2 for proton-proton collisions at $\sqrt{S} = 7$ TeV on the left and $\sqrt{S} = 14$ TeV on the right for two different values of the Higgs mass. These plots confirm the conclusion of Ref. [17] that corrections to the NLO rapidity distribution due to finite top mass effects are below 5%.

3 Conclusions and Outlook

We have reported on an extension of high-energy factorisation to rapidity distributions. The result has been made possible by exploiting the duality which relates DGLAP and BFKL evolution equations, to re-express high energy factorisation in terms of standard collinear factorisation. As a first application, we have performed the resummation of the rapidity distribution for Higgs production in gluon-gluon fusion. More interesting application of this formalism is to Drell-Yan rapidity distributions, which will be explored at very small x at the LHC.

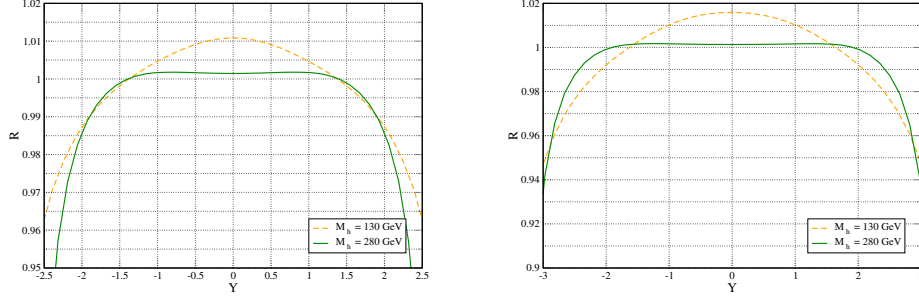


Figure 2: The ratio R as defined in Eq. (11) for two different values of the Higgs mass $m_H = 130, 280$ GeV. The plot on the left is for $\sqrt{S} = 7$ TeV, while the one on the right for $\sqrt{S} = 14$ TeV

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